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An Analytical Framework for the Assessment of Goals and Approaches by the Iraqi Airways Company

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Abstract: For the purpose of assessing Iraqi Airways Company's goals and tactics, we presented a mathematical model in this article. We began by looking at Iraqi Airways' timetables, which provide many departure cities for each route. The optimal route for the airline was then determined by using certain fuzzy integrals.

Keywords: Iraqi Airways', Assessment of Goals and Approaches

Introduction

One of the most potent and versatile functions in the domain of aggregation operators is fuzzy integrals with regard to fuzzy measures, which include generalised measures [1], non-additive measures [2], capacity [3], and [4]. The generalisations of the Choquet and Sugeno integrals are the most famous fuzzy integrals [5]. Fuzzy integrals with regard to fuzzy measures have many applications in various domains including science, engineering, economics, etc., since they rely on fuzzy measure, one of the most fundamental ideas in mathematics.

Research on airline transport is extensive, with the majority of studies focusing on service quality. The combined performance of characteristics was evaluated using fuzzy integral by [6]. To determine how service quality affects passengers' decisions while choosing an airline, [7] used fuzzy integral.When dealing with domestic airlines' in-flight service quality and uncertainty, [8] used the Fuzzy-grey approach.Applying the management insights presented by [9] to the competitive air transport industry, where each airline firm is modelled as a player, is a viable option. The use of the Sugeno integral as an aggregation operator was also explored by [5].

To choose the best town for passengers from the departing city, we provide a mathematical model that deals with Iraqi Airways, an airline, and uses many fuzzy integrals, including Sureno, Shilkrit, Choquet, Pan, and Lehrer integrals. The following is the outline of the paper. In Section 2, we define fuzzy integrals and fuzzy measures and provide an overview of their fundamental notions. Iraqi Airways Company's mathematical model for customer goals and tactics is detailed in Section 3. Section 4 presents the findings from a few research instances. A few conclusions were drawn at the end of the study.

1. Basic concept:

In this part, we review several foundational ideas of fuzzy integrals and fuzzy measures (Sugeno, Shilkrit, Choquet, PAN, as well as Lehrer) that are relevant to our study.

2.1 Fuzzy measures

By substituting the weaker criterion of monotonicity for the stronger condition of additivity, fuzzy measures expand upon the classical measure. Here we will go over the definition of fuzzy measure.

Definition 1 (fuzzy measure) [1]

$2C$ is the power set of C, and let C be a nonempty set.If the following axioms are satisfied, then a set function μ: 2**O** → [0, ∞] is a fuzzy measure on (**O**, □). If i-μ(**O**) > 0, then $\mu(E) = 0$. For each A, B in 2**O**, the equality ii-$A \leq B$ implies that $\mu(A) < \mu(B)$. subject to homotopy In the literature, several unique fuzzy metrics have been proposed. A fuzzy measure, which is defined as follows, is one of them.

Definition 2 (l- fuzzy measure) [10]

Allow lÎ(-1,¥). For any pair of disjoint subsets A and B of C, a normalised set function *b* defined on 2C is referred to as a l-fuzzy measure on C. *gl* (*A* ⋃ *B*) = *gl* (*A*) + *gl* (*B*) + l*gl* (*A*)*gl* (*B*)

Fuzzy Integrals

In this part, we will define some of the fuzzy integrals with regard to fuzzy measures that are relevant to our study, including the following: Sugeno, Shilkrit, Choquet, Pan, and Lehrer integrals.

Definition 3 (The Sugeno integral) [1]

Let μ be a normalized fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \dots, a_n\}$ where $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n \leq 1$. The Sugeno integral $\int f(c).\mu(c)$ of f w. r. t. μ is defined as *n* *SI* □ (*f*) □ □ *f* . □ □ □ (*a* $_i$ □ □(□*c* | *f* (*c*) □ a_i □)) ...(1) *i* □1 Sugeno integral is defined only for functions whose range is included in [*0, 1*], and normalized fuzzy measure. The Shilkret integral is special type of Sugeno integral has been defined as follows (see [11]).

Definition 4 (The Shilkret integral) [11]

Let μ be a normalized fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \dots, a_n\}$. The Shilkret integral of a measurable function f: C→ [0, 1] is given by □*Sh*□ (*f*) □ *Max* □a_i .□ □□*c* | *f* □*c* □ □ a_i □□□□ ...(2)

a□□0,1□□where $0 \le a_1 \le a_2 \le \cdots \le a_n \le 1$,□ : $A \to [0, 1]$ is a fuzzy measure. The most natural fuzzy integral is Choquet integral, which is an extension of the ordinary integral (Lebesgue's integral). The definition of Choquet integral is as follows.

Definition 5 (The Choquet integral) [1]

Let μ be a fuzzy measure on C and f a function on C with range$\{a_1, a_2, a_3, \ldots, a_n\}$, where $0 \le a_1 \le a_2 \le \cdots \le a_n \le 1$. The Choquet integral is given by

$$(Ch)\int f \, d\mu = \sum_{i=1}^{n} (a_i - a_{i-1}).\mu(\{c \mid f(c) \ge a_i\}) \quad \ldots(3)$$

where $a_0 = 0$.

The Pan-integral is related to finite partitions of X, the definition of Pan-integral is as follows.

Definition 6 (The Pan-integral) [1], [2]

Let μ be a fuzzy measure on C .The PAN integral of a measurable function $f: C \to [0, \infty]$)

$$(p)\int_A f \, d\mu = Max \left\{ \sum_{i=1}^{n} k_i \mu(A_i) \mid \sum_{i=1}^{n} b(k_i, A_i) \le f, (A_i)_{i=1}^{n} \text{is a partition of } C \right\} \quad \ldots(4)$$

where $b(k, A)(c) = \begin{cases} k & \text{if } c \in A. \\ 0 & \text{elsewhere}. \end{cases}$

and k is constant.

The Lehrer (concave) integral recently introduced by Lehrer [12] .The following definition, gives an explicit formula for the Lehrer integral.

Definition7 (The Lehrer integral) [2], [12]

Let μ be a fuzzy measure on C .The Lehrer integral of a measurable function $f: C \to [0, \infty]$ is

$$(L)\int_A f \, d\mu = Max \left\{ \sum_{i=1}^{n} k_i \mu(A_i) \mid n \in N, \sum_{i=1}^{n} b(k_i, A_i) \le f \right\} \quad \ldots(5)$$

where $b(k, A)(c) = \begin{cases} k & if\ c \in A. \\ 0 & elsewhere. \end{cases}$

and k is constant.

The relationships among above fuzzy integrals (Shilkret, Choquet, Pan, and Lehrer) are shown in the following remark (See [2]).

Remark 1[2]:

The relationships among Shilkret(Sh),Choquet(Ch),and Lehrer(L) is Sh ≤ Ch ≤ L and the relationship between Shilkret(Sh) and Pan(P) is Sh ≤ PAN

A Mathematical model for passenger strategies in Iraqi Airways Company

Imagine a traveller who plans to use Iraqi Airways to go from city c1 to city c2, and finally to city c3, with many other possible departure cities.The first useful component to consider when building a mathematical model is the passenger's level of relevance in reaching these cities, which are measured by fuzzy variables and reflect the passenger's value in many aspects (e.g., economically, touristically, etc.). How accessible these cities are from a given departure point (i.e., the distances between the departure and first arrival cities) is another useful component of the mathematical model. A function f: $\mathbf{O} \to [0, \infty]$ will be used to indicate the combined degrees of both the significance and accessibility of travelling from a certain departure city. Since the traveller starts their journey in city c4, the most accessible city is c1, the transit city is c2, and eventually c3, we may deduce that f(c1) > f(c2) ≥ f(c3).Since the degree of significance and the values for the measure are represented using the same terminology, f(c) is similar to □(A), where A is a subset of C. Hence, a specific connection $\mu f(ci) = \mu(\{c|(c) \ge f(ci)\})$ may be defined using the values of □ and f. The significance of reaching ci and all similarly accessible cities is denoted by this expression. Therefore, if we reach ci, we can also reach all cities with a higher probability than ci (i.e., all cities in the set $\{c|f(c) \ge f(ci)\}$), and the passenger achieves $\mu f(c\text{i})$ when they decide to reach ci. For any $A \leq \mathbf{O}$, these degrees are represented by the fuzzy measure □(A). By definition, this imprecise metric is monotonically increases and has bounds; due to the boundary criteria, reaching cities is not significant. Possible methods for each city ci should take into account both its accessibility f(ci) and its significance $\mu f(c\text{i})$. Iraqi Airways' potential methods for optimal performance, taking into account Iraqi airports and airways pathways in Iraq and other countries, are examined in this study via the lens of fuzzy measurements and fuzzy integrals.

This goal is accomplished by using fuzzy integrals such as Sugeno, Shilkret, Choquet, PAN, and Lehrer. These integrals are used to combine the relevance and reliability of certain research instances involving Iraqi airways, as will be shown in the next section.

4-Studies of individual cases

Here we look at a few case studies that were extracted from Iraqi Airways timetables in 2016 for the purpose of this section.

Case 1 (Baghdad–Basrah–Dubai) and the arbitrary departure city Sulaimaniya or Mosul are the mentioned instances.

instance 2 (Baghdad–Erbil–Dubai), with basrah or Najaf as an arbitrary departure city. arbitrary departure city Mosul or Tikrit, and case 3 (Erbil-Sulaimaniya-Dubai).

case4 (Erbil–Amman), and optionally Mosul or Sulaimaniya as the departing city.

instance 5 (Basrah–Amman), with the option to randomly leave from Najaf or Nasriyah. examples 1 and 2 are covered extensively here, whereas Table-1 displays the other examples.

First Case Study: Travelling from Sulaimaniya to Baghdad via Basrah and finally Dubai.(Dubai →

Basrah → Baghdad)

Starting from Sulaimaniya, we will go to Baghdad, Basrah, and Dubai. The following table displays the values of fuzzy measures (importance factor) for reaching cities {c1, c2, c3}.

Table 1: Fuzzy metrics for attaining {c1, c2, c3}

set	$\{c_1\}$	$\{c_2\}$	$\{c_3\}$	$\{c_1,c_2\}$	$\{c_1,c_3\}$	$\{c_2,c_3\}$	$\{c_1,c_2, c_3\}$
□□	0.7	0.4	0.5	0.86	0.9	0.73	1

We used □-fuzzy measures, with □ = -0.84, to get {c1, c2},{c1, c3},{c2, c3},{c1, c2, c3}.

You can see the different levels of accessibility from Sulaimaniya city in the table below.

Table 2-Grades of accessibility from Sulaimaniya city

set	c_1	c_2	c_3
f	0.9	0.8	0.3

The following table shows the relative importance of each city.
Table 3-Ratings of c1, c2, and c3's significance

set	c_1	c_2	c_3
μ_f	0.7	0.86	1

To apply Sugeno integral, we shall use a permutation on $f(c_i)$. That is $f(c_3) \le f(c_2) \le f(c_1)$ $(0.3 \le 0.8 \le 0.9)$.

To find μ_f, we use the formula $\mu_f(\{c_i\}) = \mu(\{c \mid f(c) \geq f(c_i)\})$:

$\mu_f(\{c_3\}) = \mu(\{c \mid f(c) \geq f(c_3)\}) = 1$

$\mu_f(\{c_2\}) = \mu(\{c \mid f(c) \geq f(c_2)\}) = \mu\{c_1, c_2\} = 0.86$

$\mu_f(\{c_1\}) = \mu(\{c \mid f(c) \geq f(c_1)\}) = \mu\{c_1\} = 0.7$

Now, we apply Sugeno integral (equation (1)) with $a_i = f(c_i)$, for this case we obtained

$$SI_\mu(f) = Max\,\{\min\{f(c_1), \mu_f(\{c_1\})\}, \min\{f(c_2), \mu_f(\{c_2\})\}, \min\{f(c_3), \mu_f(\{c_3\})\}\}$$
$$= Max\,\{\min\{0.9, 0.7\}, \min\{0.8, 0.86\}, \min\{0.3, 1\}\}$$
$$= Max\,\{0.7, 0.8, 0.3\} = 0.8$$

By using Shilkrit integral (equation (2)), we obtain

$$Sh_\mu(f) = Max\{f(c_1).\mu_f(\{c_1\}), f(c_2).\mu_f(\{c_2\}), f(c_3).\mu_f(\{c_3\})\}.$$
$$= Max\{(0.9)(0.7), (0.8)(0.86), (0.3)(1)\} = 0.688$$

If we apply Choquet integral (equation (3)) with $a_i = f(c_i)$, we obtain

$$(Ch)\int f d\mu = \sum_{i=1}^{n} (a_i - a_{i-1}).\,\mu(\{c \mid f(c) \ge a_i\})$$

$$(Ch)\int f d\mu = (0.3-0)(1) + (0.8-0.3)(0.86) + (0.9-0.8)(0.7)$$
$$=0.3+0.43+0.07$$

$$=0.8$$

Also we can apply Pan–integral using equation (4), we get

□□*f* □c_2□.□ □□c_1,c_2□□ □ *f* □c_3□.□ □□c_3□□, *f* □c_3□.□ □□c_1,c_3□□ □ *f*

□c_2□.□ □□c_2□□,□□□□

(*p*)□$_A$ *fd* □ □ *Max* □*f*□ □c_3 □.□ □□c_2 ,c_3□□ □ *f* □c_1 □.□ □□c_1□□, *f* (c_3).□(□*C* □),

□□□□

□□*f* □c_1□.□ □□c_1□□ □ *f* □c_2□.□ □□c_2□□ □ *f* □c_3□.□ □□c_3□□ □□□

$= Max$ □0.838, 0.59, 0.849, 0.3,1.1□□

□ 1.1

Finally, we can apply Lehrer–integral using equation (5), we obtain

□f □c2 □.□ □□c1 ,c2 □ □ f □c3 □.□ □□c3 □ □ (f □c1) □ f (c2)□.□ □□c1□□, □□

□□ □

□f □c3 □.□ □□c1 ,c3 □ □ f □c2 □.□ □□c2 □ □ (f □c1) □ f (c3)□.□ □□c1□□, □□

(L) A□ fd □ □ Max □□□f□ □c3 □.□ □□c2 ,c3□ □ f □c1 □.□ □□c1□ □ (f □c2) □ f (c3)□.□ □□c 2□□,

□□□□□

□□(ff ((cc3).).□□(□□□Cc □□)□ □ □ □ ff ((cc 2).) □□ □f □(cc 3□)□□. □□ (f□ c□c2

□□). □□ □□□f c(c□1□) □ f (c3)□.□(□c1□),□□□□

Imagine now that we change the starting city but otherwise follow the same route.Consider Mosul as the starting point for a journey to Baghdad, Basrah as a stopover, and Dubai as the final destination.
(Dubai to Basrah via Baghdad)
the starting point is Mosul, c1: Baghdad, c2: Basrah, and c3: Dubai You can see the different levels of accessibility from Mosul city in the table below.

Table 4: Levels of Accessibility from Mosul City

set	c_1	c_2	c_3
g	0.8	0.7	0.1

The following table shows the relative importance of each city.
Table 5: Levels of significance for every city

set	c_1	c_2	c_3
μ_g	0.7	0.86	1

To apply Sugeno integral, we shall use a permutation on $g(c_i)$. That is
$g(c_3) \le g(c_2) \le g(c_1)$ $(0.1 \le 0.7 \le 0.8)$. To find μ_g,
we use the same previous method,

$\mu_g(\{c_3\}) = 1$

$\mu_g(\{c_2\}) = \mu(\{c_1, c_2\}) = 0.86$

$\mu_g(\{c_1\}) = \mu(\{c_1\}) = 0.7$

First, we apply Sugeno integral (equation (1)) with $a_i = g(c_i)$, for this case $SI_\mu(g)$ $= Max\,\{0.7, 0.7, 0.1\} = 0.7$

By using Shilkrit integral (equation (2)), we obtain

$Sh_\mu(f) = Max\,\{0.56, 0.602, 0.1\} = 0.602$

If we apply Choquet integral (equation (3)) with $a_i = g(c_i)$, we get

$(Ch)\int g d\mu = 0.1+0.516+0.07 = 0.686$

Also we can apply Pan-integral (equation (4)), we obtain

Lastly, by using the Lehrer integral (equation (5)), we can see that the maximum value of (L) □ gd □ is 0.89, with values of 0.722, 0.86, 0.873, 0.83, and 0.89.

For Case 2, we may use the same formulas as for Case 1 to determine the fuzzy integrals (Sugeno, Shilkret, Choquet, PAN, Lehrer). Starting at Najaf as the starting point and working our way to Baghdad, Erbil, and finally Dubai. The route leads from Baghdad to Erbil and then to Dubai.
Starting from Najaf, we will go to Baghdad, Erbil, and Dubai (c1–c3).
The following table displays the values of fuzzy measures (importance factor) for reaching cities {c1, c2, c3}.
The important factor for fuzzy measurements in reaching cities {c1, c2, c3} and their values are shown in Table 6.

set	$\{c_1\}$	$\{c_2\}$	$\{c_3\}$	$\{c_1,c_2\}$	$\{c_1,c_3\}$	$\{c_2,c_3\}$	$\{c_1,c_2, c_3\}$
□□	0.5	0.3	0.7	0.68	0.92	0.83	1

Using the □-fuzzy measure, with □ = -0.8, we were able to get the same result as before. The following table displays the degrees of accessibility from Najaf city.
Levels of accessibility from Najaf city, as shown in Table 7.

set	c_1	c_2	c_3
f	0.8	0.3	0.1

The following table shows the relative importance of each city.
Table8: Levels of significance for all cities

set	c_1	c_2	c_3
□$_f$	0.5	0.68	1

Utilising a permutation on f (ci), we will implement the Sugeno integral. The expression is as follows:
$(0.1 \leq 0.3 \leq 0.8)$ where $f(c3) \leq f(c2) \leq f(c1)$. When the Sugeno Integral (equation (1)) is first applied, SI□(f) = 0.5. Equation (2), which employs the
Shilkret integral, yields $Sh\mu(f)$ = 0.4.

When the Choquet integral (equation (3)) is used, the result is (Ch)□f d□ = 0.486. Also, we may use the Pan integral, which is given by equation (4), with (p) □ fd □ □ 0.56.
At long last, we may use the Lehrer integral, which is (L)□ fd □ □0.56, in equation (5).

Imagine now that we change the starting city but otherwise follow the same route. Beginning in Basrah, we go to Baghdad, via Erbil, and finally to Dubai.
Begin at Basrah and make your way to Baghdad, Erbil, and Dubai (c1–c3). The following table displays the accessibility degrees from Basrah city.
Grasp the degree of accessibility from Basrah city in Table 9

set	c_1	c_2	c_3
g	0.5	0.4	0.2

The following table shows the relative importance of each city.
Table 10-Levels of significance for every city

set	c_1	c_2	c_3
μ_g	0.5	0.68	1

The Sugeno integral will be applied by permuting g (ci). The expression is $b(c3) < b(c2) \leq b(c1)$ for values between 0.2 and 0.5. We begin by using the Sugeno Integral, SI□(g) = 0.5. The Shirk integral yields $Sh\mu()$ = 0.272. If the Choquet integral is used, the result is (Ch)□g d□ = 0.386. Another option is to use the Pan-integral, where (p) □ gd □ □ 0.51. In conclusion, we may get (L) □ gd □ □ 0.51 by using the Lehrer integral.

The instances in Table-11, numbered 1 through 5, depict the typical aircraft routes used by Iraqi's Airways [13] for various scenarios, with the values of these routes determined by distinct fuzzy integrals.
Example cases (Table 11)

cases	The path	Departure city	Sugeno integral $SI_\mu(f)$	Shilkrit Integral $Sh(f)$	Choquet integral $(C)\int f d\mu$	Pan integral (p) $\int f d\mu$	Lehrer Integral $(L)\int f d\mu$
1	Baghdad →Basrah→Dubai	Sulaimania	0.8	0.688	0.8	1.1	1.1
		Mosul	0.7	0.602	0.686	0.89	0.89
2	Baghdad→Erbil→Dubai	Najaf	0.5	0.4	0.486	0.56	0.56
		Basrah	0.5	0.272	0.386	0.51	0.51
3	Erbil→Sulaimaniya→Du bai	Mosul	0.616	0.45	0.646	0.83	0.83
		Tikrit	0.616	0.431	0. 552	0.74	0.74
4	Erbil→Amman	Mosul	0.6	0.54	0.62	0.7	0.7
		Sulaimania	0.6	0.42	0.54	0.66	0.66
5	Basrah→ Amman	Nasriyah	0.7	0.56	0.65	0.83	0.83
		Najaf	0.5	0.4	0.47	0.71	0.71

The following outcomes may be inferred from Table-11.
When we utilised the Sugeno integral to choose two random departure cities for the same airline route in examples 2, 3, and 4, we got the same findings. However, in case 1, 5, Sulaimaniya and Nasriyah were the best choices for the recommended routes. For the Shilkret integral, the result is the same.

Since the distances from departure cities to destinations are different, it is not surprising that we observed a discrepancy in the values of the paths when we used the Choquet integral, which focuses on the degree of accessibility. The integrals of Pan and Lehrer are also equivalent. As can be seen in Table-11, our findings are in agreement with Remark1 for all instances under consideration.

Conclusions

For the purpose of assessing Iraqi Airways' goals and tactics, we have presented a mathematical model in this article.To find the most cost-effective route, Iraqi's Airways has used fuzzy integrals (Sugeno, Shilkret, Choquet, Pan, Lehrer) in various scenarios with optional departure cities. Conforming to the actual outcomes on the realistic front, the results were discovered. Future plans may be explored using this model, which takes into account the optimal routes for airlines and the capacity of fuzzy integrals to handle difficulties with departure and transit city selection for Airways

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