

JULIA AND MANDELBROT SETS FOR SOME TWO-DIMENSIONAL QUADRATIC REFLECTIONS**Kutlimuratov Ravshanbek Rozboeich**

Teacher of Tashkent State University of Economics

ravishbektdiutf@gmail.com*Received: Feb 22, 2024; Accepted: March 29, 2024; Published: Jun 8, 2024;*

Abstract: In this article, in the study of two-dimensional quadratic reflection, graphic analysis and phase portrait are important tools for the study of dynamic systems, and the role of this method is discussed. Also, in order to contribute to the development of the industry, various comments and suggestions were presented in the article.

Keywords: Graphic Analysis, Phase Portrait, Point Movement, Trajectory, Dynamic System, Production.

This is an open-access article under the [CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/) license**Introduction**

Graphical analysis involves presenting data graphically and then analyzing it. One of the simplest and most informative graphs possible is the line graph. A phase portrait is a graphical representation of how the quantities describing the state of the system (also called dynamic variables) are related to each other. This allows us to visualize the behavior of the system in time and space.

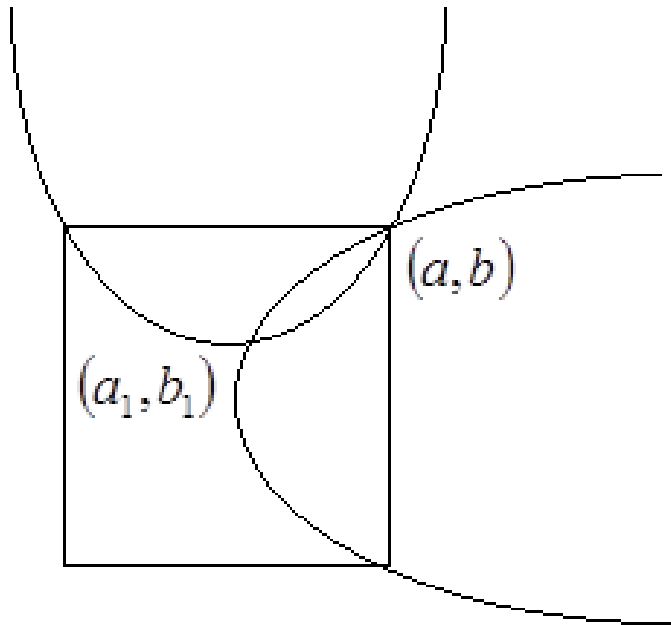
From the point of view of mathematics, any dynamic system describes the movement of a point in space, which is a spatial trajectory. The study of phase portraits of real objects showed that this method can be used for comparative production of the stability of production processes. To build a phase portrait of the system, it is necessary to build a vector field of the directions of the system trajectories at each point of the x, y plane. This is a difficult task, so a qualitative approach is used.

It should be noted that graphical analysis and phase portraits are only one of many tools used to study dynamical systems, and their application may vary depending on the specific system and problem.

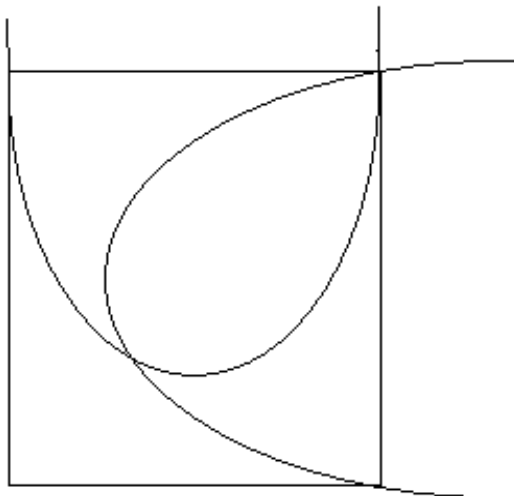
Results and Discussion

The method of graphic analysis shows that $|x| \leq a, |y| \leq b$ all points that do not belong to the rectangle tend to infinity, $|x| \leq a, |y| \leq b$ all points belonging to a rectangle (a, b) tends to the point. So $|x| \leq a, |y| \leq b$ rectangle (a, b) it is the pool of a fixed point. This fixed point is a hyperbolic fixed point because the left side is the attractor and the right side is the pusher. $|x| \leq a, |y| \leq b$ by checking the proimage of a rectangle graphically, we find that it is inside itself, and from this we can draw a conclusion that the Julia set is connected. Now $y = x^2 + c_2$ and $x = y^2 + c_1$ let's check the method of graphic analysis when the parabolas have two points in common. Intersection points of parabolas (a, b) and (a_1, b_1) let these points be $Q_{c_1 c_2}$ there will be fixed points for reflection. Conditionally (a, b) we define the fixed point whose coordinates are the largest. In that

case $y = x^2 + c_2$ and $x = y^2 + c_1$ we can describe parabolas as follows.



It can be seen from the graphic analysis method $|x| \leq a, |y| \leq b$ all points that do not belong to a rectangle tend to infinity, and those that do (a_1, b_1) tends to a fixed point. (a, b) and the fixed point remains receding.



Even from this case (a_1, b_1) pool of a fixed point (a, b) besides the point $|x| \leq a, |y| \leq b$ consists of all points belonging to a rectangle. The biggest fixed point (a, b) we determined through this

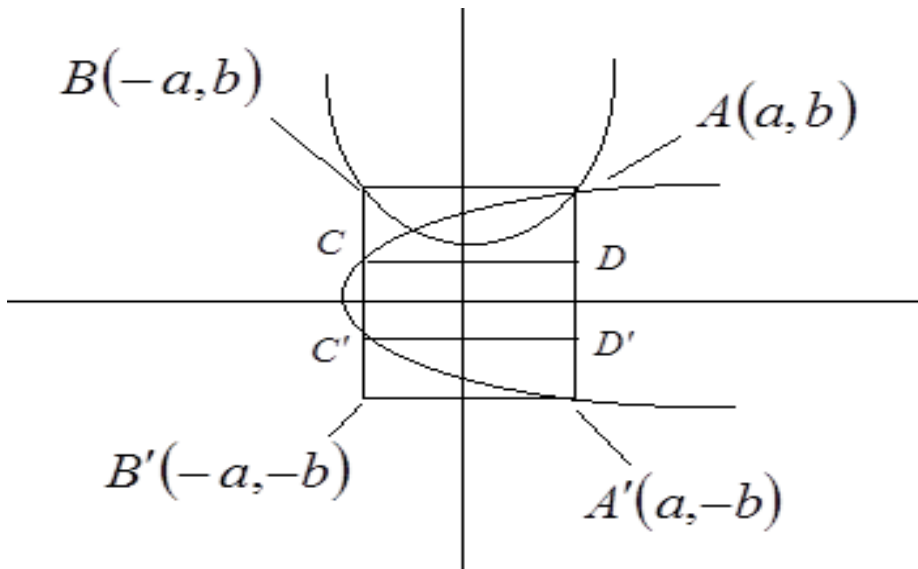
$$\begin{cases} a = b^2 + c_1 \\ b = a^2 + c_2 \end{cases} \Rightarrow \begin{cases} c_1 = a - b^2 \\ c_2 = b - a^2 \end{cases}$$

acceleration will look like this.

$$Q_{ab} = \begin{cases} x' = y^2 + a - b^2 \\ y' = x^2 + b - a^2 \end{cases}$$

But (a, b) it should be noted that the fixed point is the largest of the other fixed points. Analyzing the graph, it can be said that if the ends of the parabolas $|x| \leq a, |y| \leq b$ Julia set if it belongs to a rectangle $|x| \leq a, |y| \leq b$ rectangle itself, otherwise something else will happen. Let the end of one of the parabolas $|x| \leq a, |y| \leq b$ does not belong to a rectangle, in this case we will analyze the graph. For example $x = y^2 + a - b^2$ the tip of the parabola $|x| \leq a, |y| \leq b$ does not belong to a rectangle, in that case $|x| \leq a, |y| \leq b$ of a rectangle CD and below the straight line $C'D'$ image of all points above the straight line Q_{ab} from one iteration using reflection $|x| \leq a, |y| \leq b$ lies outside the rectangle. $|x| \leq a, |y| \leq b$ rectangle in one iteration $|x| \leq a, |y| \leq b$ the set of all points reflected outside the rectangle M_1 let us define by this set is open at the top and bottom and closed at the right and left. C, C', D, D' are the coordinates of the points

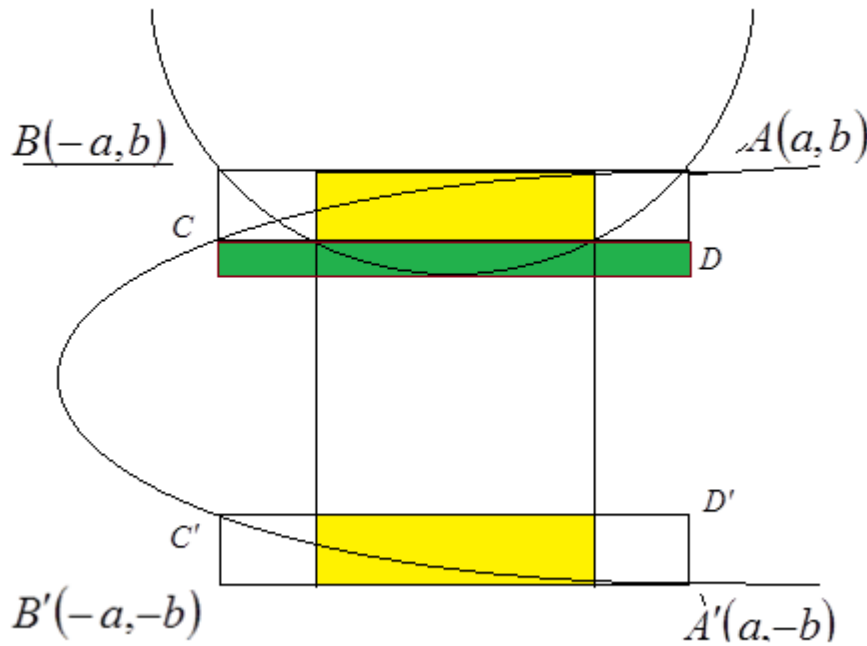
$$C(-a, \sqrt{b^2 - 2a}), C'(-a, -\sqrt{b^2 - 2a}), D(a, \sqrt{b^2 - 2a}), D'(a, -\sqrt{b^2 - 2a})$$



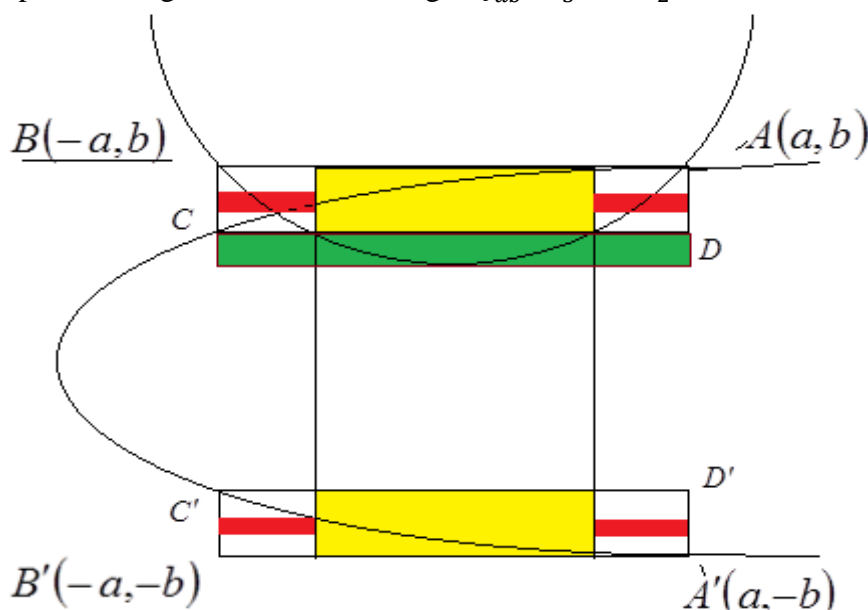
If $b - a^2 \geq -b$ and $a - b^2 \geq -a$ are the vertices of the parabolas $|x| \leq a, |y| \leq b$ belongs to a rectangle and the Julia set is one-joint, otherwise i.e. $b \geq \frac{a^2}{2}$ and $a \geq \frac{b^2}{2}$ and again accordingly $a - b^2 \geq \sqrt{b^2 - 2a}$ and $b - a^2 \geq \sqrt{a^2 - 2b}$ only if the inequalities are satisfied, there are two non-one-joint Julia sums $ABCD$ and $A'B'C'D'$ consists of rectangles. This is the reason $ABCD$ all points of a quadrilateral are reflected to itself, and its proimages are two $ABCD$ and $A'B'C'D'$ will be in rectangles. $A'B'C'D'$ all points of the rectangle $ABCD$ is reflected in the rectangle and the original image remains. So $A'B'C'D'$ all points of the rectangle in one iteration $ABCD$ moves to a rectangle and stays there for all subsequent iterations. Now $b \geq \frac{a^2}{2}$ or $a \geq \frac{b^2}{2}$ respectively with one of the defaults $a - b^2 \geq \sqrt{b^2 - 2a}$ or $b - a^2 \geq \sqrt{a^2 - 2b}$ let's analyze graphically if one of the inequalities is not fulfilled.

$a - b^2 \geq \sqrt{b^2 - 2a}$ we see the case where the inequalities are not satisfied, the second case is the same. So $a - b^2 \leq \sqrt{b^2 - 2a}$ or $-a^2 \leq \sqrt{a^2 - 2b} \Rightarrow b^4 - 2ab^2 + 2b \leq 0, a^4 - 2a^2b +$

$2a \leq 0$ let it be. In that case, if we analyze the graph, inside the rectangles $ABCD$ and $A'B'C'D'$ two rectangles painted in yellow in the picture are in one iteration $CC'DD'$ is reflected inside the rectangle and in the second iteration $|x| \leq a, |y| \leq b$ goes out of the rectangle and then tends to infinity, we $|x| \leq a, |y| \leq b$ in two iterations of the rectangle $|x| \leq a, |y| \leq b$ the set of all points outside the rectangle M_2 if we define it as $Q_{ab}(M_2) \subset M_1$ will be.



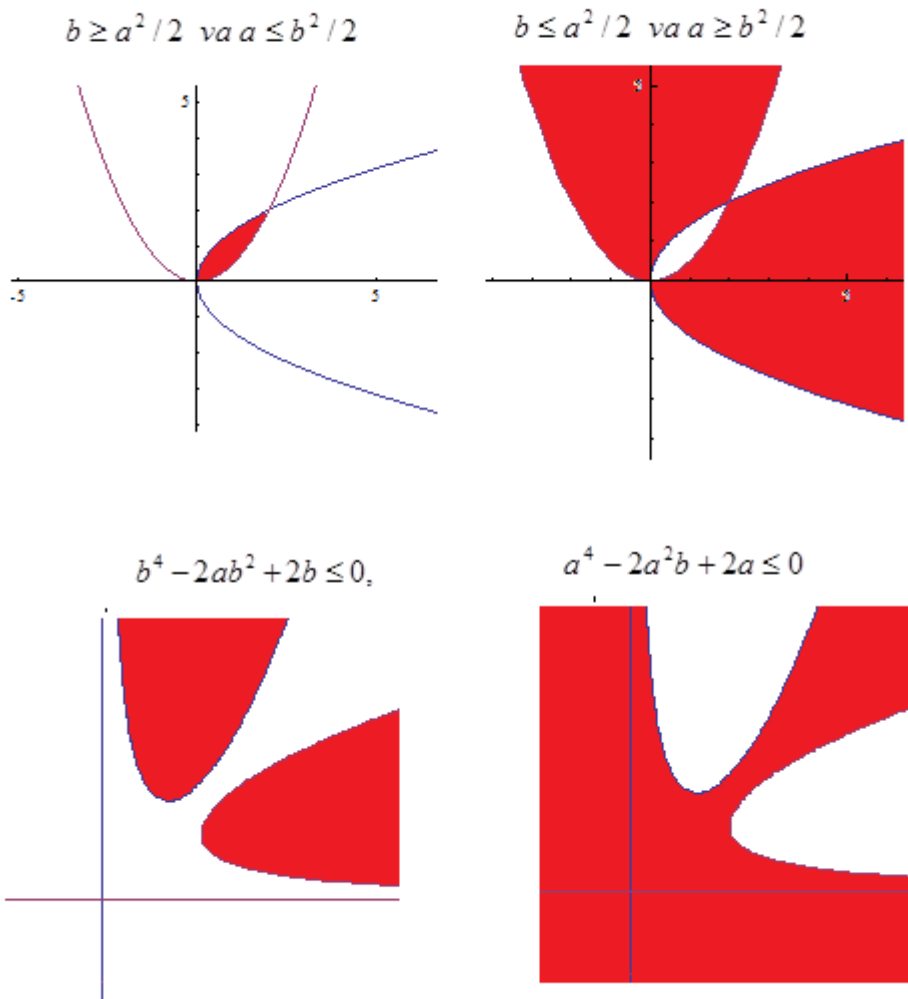
By graphical analysis, we find that the prototype of M_2 that is, the yellow rectangles, is the red rectangles in the four small pictures. The points inside these red rectangles are reflected to the yellow rectangle above in one iteration, to the green rectangle in the second iteration, and to the green rectangle in the third iteration. $|x| \leq a, |y| \leq b$ goes out of the rectangle and then tends to infinity, we $|x| \leq a, |y| \leq b$ to 'g'ri to 'rtburchakning uchta iteratsiyada $|x| \leq a, |y| \leq b$ if we define as the set of all points that go outside the rectangle $Q_{ab}(M_3) \subset M_2$ will be.



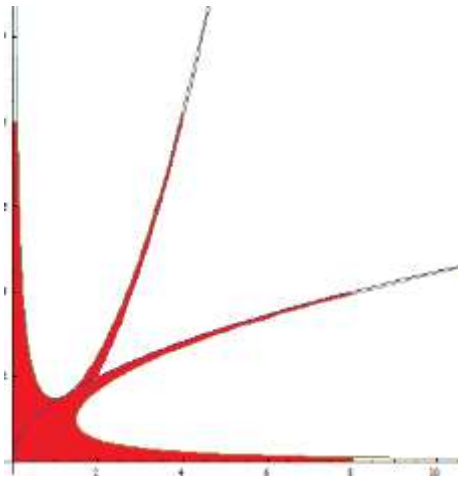
We continue this process infinitely and in n iterations $|x| \leq a, |y| \leq b$ the set of all points reflected outside the rectangle M_n if we define it as $Q_{ab}(M_n) \subset M_{n-1}$. After doing this process a few times $|x| \leq a, |y| \leq b$ belongs to a rectangle Q_{ab} the question arises whether there are any remaining points in this rectangle through reflection. The answer is yes, even if there are infinitely many points. For example $|x| \leq a, |y| \leq b$ fixed points, periodic points, quasi-fixed and quasi-periodic points belonging to a rectangle, even if it is accelerated infinitely many times $|x| \leq a, |y| \leq b$ remains a rectangle. The nature of this set of points is equivalent to the nature of the Cantor set. This is a collection of points Λ if we define by .

$$\Lambda = \{|x| \leq a, |y| \leq b\} \setminus \sum_{i=1}^{\infty} M_i$$

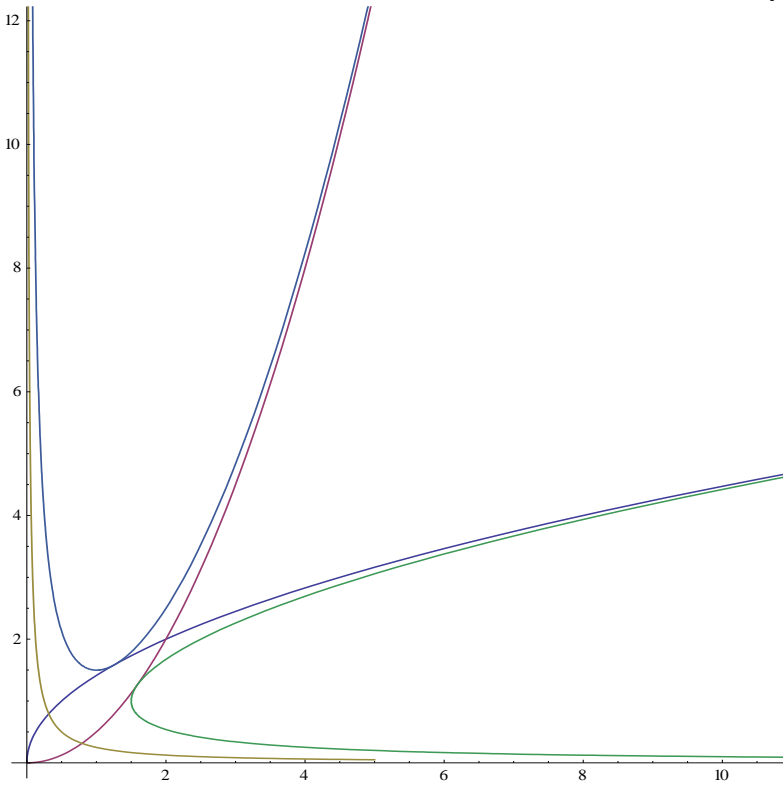
Now $b \geq \frac{a^2}{2}$ and $a \leq \frac{b^2}{2}$ and again $b^4 - 2ab^2 + 2b \leq 0, a^4 - 2a^2b + 2a \leq 0$ let's talk about what the area bounded by curves means.

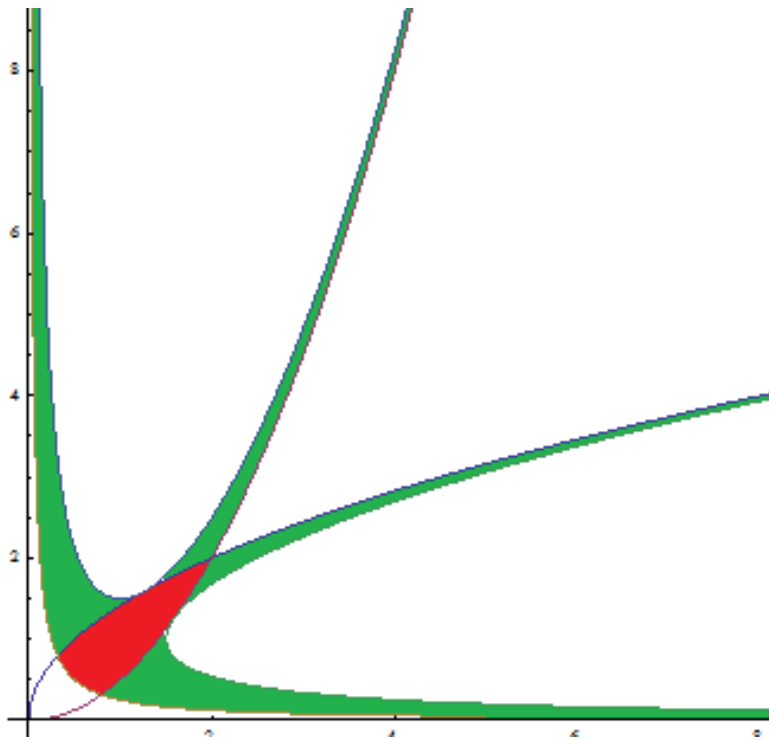


If we see both in one coordinate system



and this from the bottom $4ab = 1$ if we limit ourselves to hyperbola





This gives us an approximate view of the Mandelbrot set we need.

To sum up, modern fields such as exact sciences, information, mathematical economy, Internet and digitization are developing consistently in the world today. Of course, keeping up with the times, it is necessary to ensure the development of the above industries in our country in order to develop and improve our economy. Because with this, we not only contribute to the development of our country's economy, but also contribute to the well-being of the population and the peace of our country. Now you “JULIA and MANDELBROT sets for some two-dimensional quadratic reflections” once you have a better understanding of them, you have the opportunity to apply them in your economic activity.

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