EJHEAA
ISSN : 3032-1123
https://doi.org/10.61796/ejheaa.v1i6.590

GRAPHICAL ANALYSIS FOR SOME TwO DIMENSIONAL DYNAMICAL SYSTEMS

Islamova M

Received: Feb 22, 2024; Accepted: March 29, 2024; Published: Jun 10, 2024;

Abstract:
Ba'zi ikki o'lchovli dinamik sistemalar uchun grafik tahlil usuli Ushbu maqolada ba'zi ikki o'lchovli dinamik sistemalar uchun grafik tahlil usuli kiritilgan. Grafik tahlil usuli yordamida berilgan akslantirishlar uchun Julia va Mandelbrot to'plamlarining ayrim xossalari tahlil qilingan.

Keywords: Method of graphical analysis; Julia set; Mandelbrot set.

Introduction

There exist in [2],[3], [4], [5],[6], [7] method graphical analysis for one dimensional dynamical systems. Suppose we have the graph of a function F and wish to display the orbit of a given point x_0. We begin by superimposing the diagonal line $y = x$ on the graph of F. The points of intersection of the diagonal with the graph give us the fixed points of F. To find the orbit of x_0, we begin at the point (x_0, x_0) on the diagonal directly above x_0 on the x axis. We first draw a vertical line to the graph of F. When this line meets the graph, we have reached the point $(x_0, F(x_0))$. We then draw a horizontal line from this point to the diagonal. We reach the diagonal at the point whose $y-$ coordinate is $F(x_0)$, and so the $x-$coordinate is also $F(x_0)$. Thus we reach the diagonal directly over the point whose $x-$ coordinate is $F(x_0)$, the next point on the orbit of x_0 Fig 1.

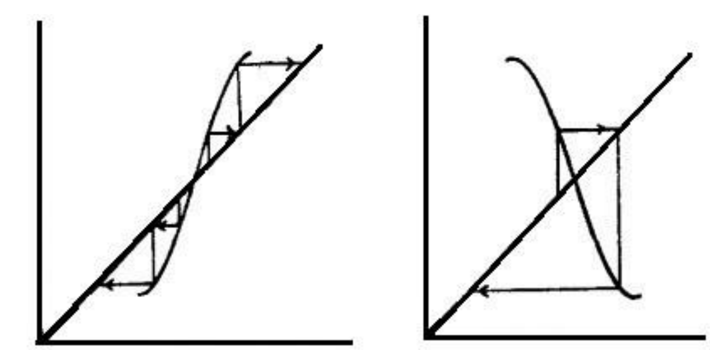

Figure. 1.

The Method of graphical analysis

In this paper, we introduce a geometric procedure that will help us understand the dynamics of some two-dimensional mappings. This procedure enables us to use the graphs of functions to determine the behavior of orbits in many cases. Suppose we have the two-dimensional mapping

$$F_{c_1 c_2}: \begin{cases} x' = f(y, c_1), \\ y' = g(x, c_2). \end{cases} \tag{1}$$

and wish to display the orbit of a given point (x_0, y_0). We begin by superimposing the graph of $x = f(y, c_1)$ on the graph of $y = g(x, c_2)$. The points of intersection of the graph $x = f(y, c_1)$ with the graph of $y = g(x, c_2)$ give us the **fixed points** of $F_{c_1 c_2}$. To find the orbit of (x_0, y_0), we begin at the point (x_0, y_0) on the XOY plane. We first draw a horizontal line to the graph of $x = f(y, c_1)$. When this line meets the graph of $x = f(y, c_1)$, we have reached the point $(f(y_0, c_1), y_0)$ then draw a vertical line and denote it by V_1. We again begin at the point (x_0, y_0) on the XOY plane we draw a vertical line to the graph of $y = g(x, c_2)$. When this line meets the graph of $y = g(x, c_2)$, we have reached the point $(x_0, g(x_0, c_2))$ then draw a horizontal line and denote it by H_1. The intersection point of V_1 and H_1 is $(f(y_0, c_1), g(x_0, c_2)) = (x_1, y_1)$ the next point of the orbit of given point (x_0, y_0). To display the orbit of (x_0, y_0) geometrically, we thus continue this procedure over and over, in the next step we denote V_{i+1} instead of V_i and H_{i+1} instead of H_i. The intersection point of V_i and H_i is the i th point of the orbit of (x_0, y_0) by the mapping of $F_{c_1 c_2}$.

Example 1.

$$F_{c_1 c_2}: \begin{cases} x' = y^2 + c_1, \\ y' = x^2 + c_2. \end{cases} \tag{2}$$

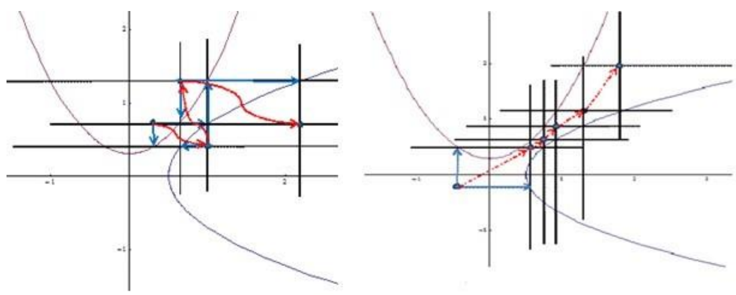

Figure. 2.

Example 2.

$$F_{c_1 c_2}: \begin{cases} x' = y^3 + c_1, \\ y' = x^3 + c_2. \end{cases} \tag{3}$$

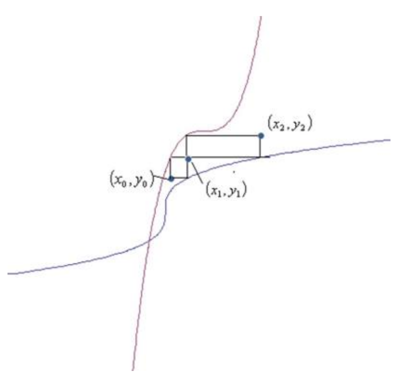

Figure. 3.

The sets of Julia and Mandelbrot

Definition The filled Julia set $K(F_{c_1c_2})$ of a mapping (1) is defined as the set of all points (x, y), that have bounded orbit with respect to mapping (1).

$$K(F_{c_1c_2}) = \{(x, y) : F^n_{c_1c_2}(x, y) \nsim \infty \ as \ n \to \infty\}$$

Definition The Julia set is the common boundary of the filled Julia set

$$J(F_{c_1c_2}) = \partial K(F_{c_1c_2}).$$

Definition The critical points of the mapping (1) are all points (x_c, y_c) which determinant of Jacobian matrix at these points is equal to zero $\Delta(J(F_{c_1c_2}(x_c, y_c))) = 0$.

Definition The Mandelbrot set $M_{F_{c_1c_2}}$ for the mapping (1) is the set of all points (c_1, c_2) on the parameter plane, which the orbits of the all critical points are bounded.

By the method of the graphical analysis, we obtain the following theorems.

Theorem 1. If the mapping (3) has only one fixed point (x^*, y^*) then this fixed point repeller and filled Julia set consists of only one point $K(F_{c_1c_2}) = (x^*, y^*)$. See Fig. 4.

At the left Mandelbrot set and at the right filled Julia set in the figures 4-8 for corresponding values of (c_1, c_2).

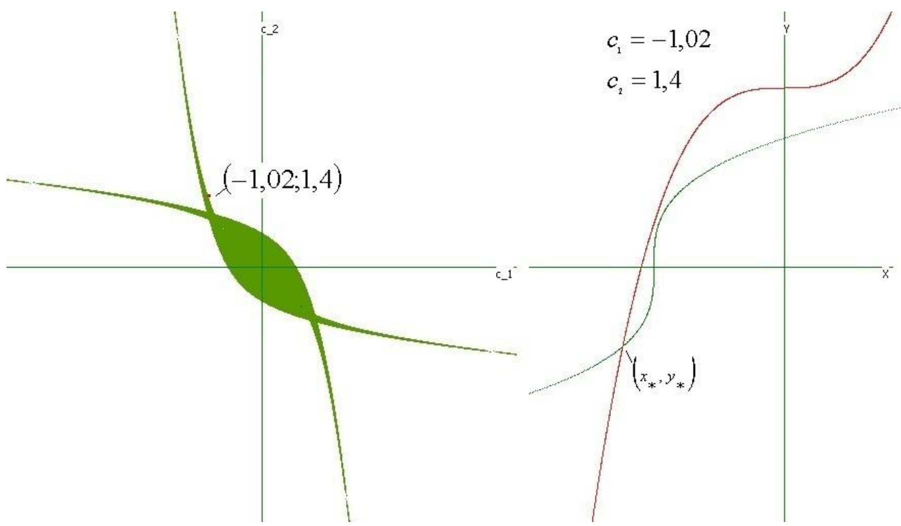

Figure. 4.

Theorem 2. If the mapping (3) has fixed points more than one (x_1^*, y_1^*), (x_2^*, y_2^*),...,(x_n^*, y_n^*) and let the points having a maximum and minimum coordinate abscissa of them (x_{max}, y_{max}^*) and (x_{min}^*, y_{min}^*) then filled Julia set consists of rectangle that vertices (x_{max}^*, y_{max}^*) and (x_{min}^*, y_{min}^*).

See Fig. 5.,Fig. 6.,Fig. 7. The black rectangles are filled Julia sets for (3) different values of (c_1, c_2).

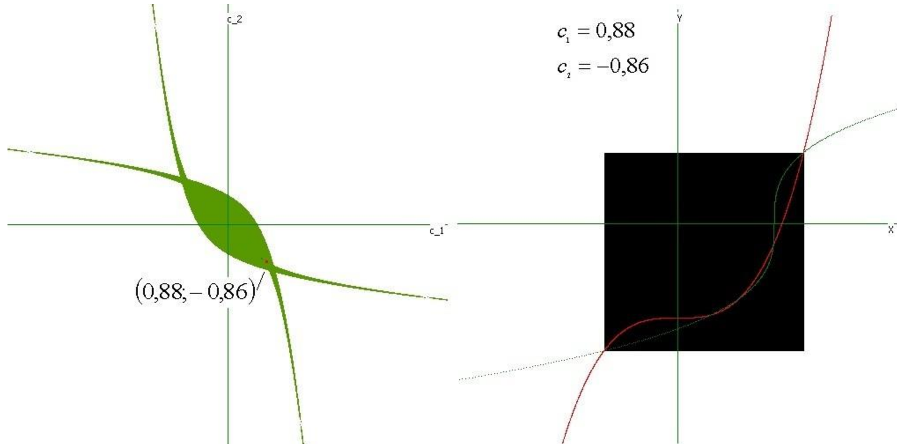

Figure. 5.

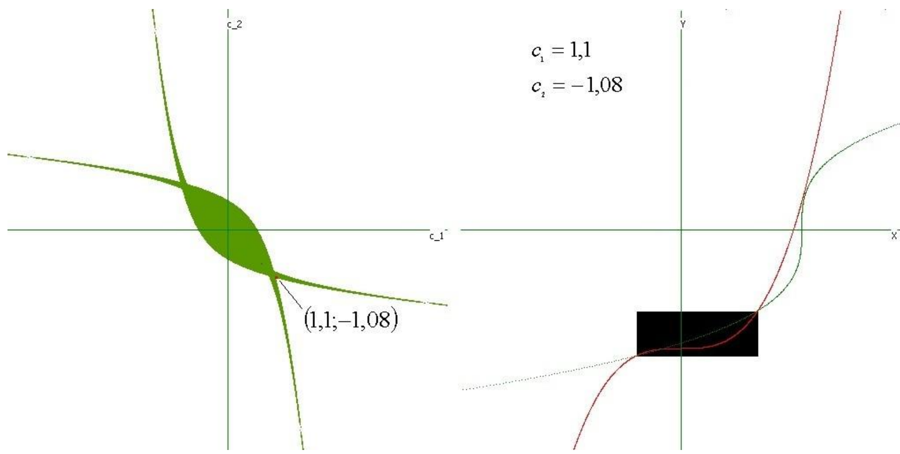

Figure. 6.

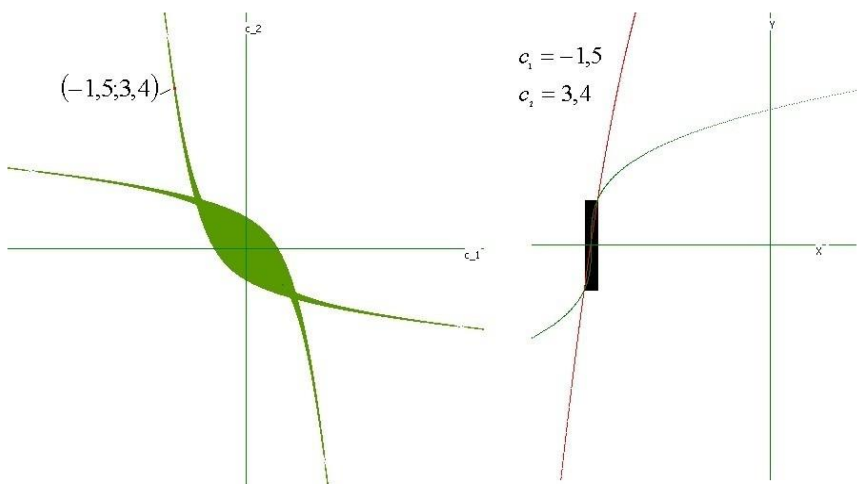

Figure. 7.

Theorem 3. The union of basins of all attractive fixed and periodic points with period two is equal to filled Julia set.

Theorem 4. The orbits of all points of filled Julia set tend to an attractive fixed point or periodic cycle with period two. It means the orbits of all points of filled Julia set are **regular**.

Theorem 5. If $(c_1, c_2) \in M_{F_{c_1 c_2}}$ for the mapping (3) then filled Julia set connected. If $(c_1, c_2) \notin M_{F_{c_1 c_2}}$ for the mapping (3) then filled Julia set is consists of only one point.

The filled Julia set depicted in Fig. 8. for $c_1 = 0, c_2 = 0$.

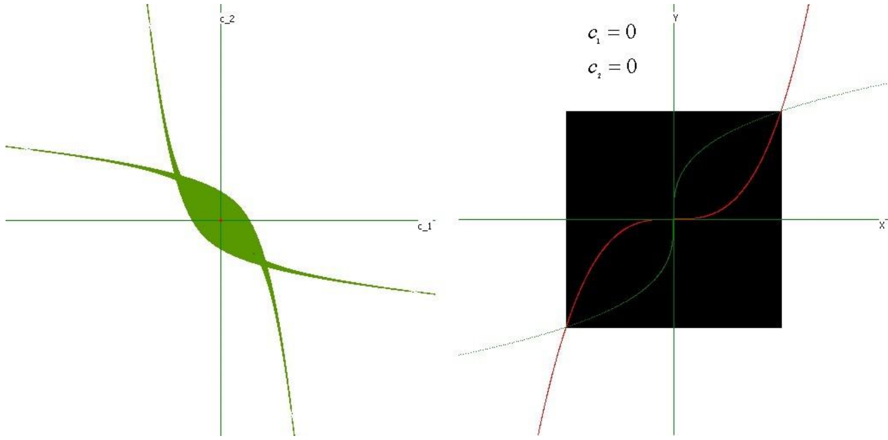

Figure. 8.

Theorem 6. Let the points $(p_1, q_1), (p_2, q_2), \ldots, (p_n, q_n)$ are the fixed points for the mapping (3), if $i \neq j$ the points (p_i, q_i) (p_j, q_j) are arbitrary two points of them, the points (p_i, q_j) and (p_j, q_i) are periodic points with prime period two.

Proof. The points are $(p_i, q_i) \neq (p_j, q_j)$ fixed, let

$$x_0 = p_i,$$

$$y_0 = q_j$$

next point of the orbit of (x_0, y_0)

$$x_1 = q_j^3 + c_1 = p_j,$$

$$y_1 = p_i^3 + c_1 = q_i$$

hence

$$x_2 = q_i^3 + c_1 = p_i,$$

$$y_2 = p_j^3 + c_2 = q_j$$

We see $(p_i, q_j) \to (p_j, q_i) \to (p_i, q_j)$ by (3). The theorem is proved. Q

Proposition. We know from [1] that the equations for finding fixed points and for finding periodic points with period two are the same, therefore this theorem is true. In the first section, we have learned the properties of fixed points, many of them are true for the periodic points with a period two.

References

[1] Ganikhodzhayev R. N., Narziyev N. B., Seytov Sh. J. Multi-dimensional case of the problem of Von Neumann
- Ulam. *Uzbek Mathematical Journal*, 3(1), 2015, pp. 11–23.

[2] Devaney R. L. A First Course In Chaotic Dynamical Systems: Theory And Experiment. Boston, 1992.

[3] Devaney R. L. An Introduction to Chaotic Dynamical Systems. New York, 1989.

[4] Feigenbaum M. J. Qualitative Universality for a Class of Nonlinear Transformations. *J. Stat. Phys.*, 19(1),1978, pp.25–52.

[5] Alligood K. T., Sauer T. D, Yorke J. A. Chaos: An Introduction to Dynamical Systems. Berlin, 2000.

[6] Easton R. W. Geometric methods for discrete dynamical systems. Oxford, 1998.

[7] Sharkovsky A. N. Coexistence of Cycles of a Continuous Map of a Line into itself. *Ukrain J. Math.*, 16(3),1964, pp.61–71.