

CALCULATION OF THE STRESS-STRAIN STATE OF CYLINDRICAL SHELLS INTERACTING WITH THE SOIL

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Abstract: The work deals with issues related to calculations with the calculation of the stress-strain state of circular cylindrical shells interacting with the soil. The results of calculating the distribution of soil pressure along the perimeter of pipelines are presented

Keywords: Cylindrical shells, soil, numerical calculation, stress-strain state, calculation algorithm, variation-difference method

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Introduction

In industry and technology, especially in connection with the creation of new modern structures, structures whose constituent or main load-bearing elements are cylindrical shells of large diameters are widely used. In many cases, the operating conditions of such structures pose challenges for designers related to calculations of the strength and stability of shells during their interaction with both the elastic filler and the surrounding elastic material. For example, with an increase in the height of berthing or fencing structures, the versatility of the shells is confirmed by the possibility of their successful use in construction "in the water" and "dry" on almost any soil foundation. This, along with calculations related to determining the settlement of the foundation soil, also requires the development of methods for determining the stress-strain state (SSS) and the stability of shells interacting with the environment (internal and external backfill). In the technical literature there are methods for calculating pipelines that take into account the unevenness of soil pressure based on the representation of the soil thickness in the form of an elastic medium, but to date there are no recommendations that would allow obtaining a sufficiently reliable picture of the distribution of soil pressure along the perimeter of pipelines - cylindrical shells of large diameters.

Works devoted to the analysis of stress-strain state of underground pipelines belong to Galerkin B.G., Borodavkin P.P., Vinogradov S.V., Kleina G.K., which sets out the principle of

consistent complication of pipeline models, describes models of pipelines and soil masses. The works of Ainbinder A.B., Kamershtein A.G. [], Akselrad E.A., Ilyin V.P., Aleshin V.V., Vislobitsky P.A., Gaiduk V.F., are devoted to calculations of the strength of underground pipelines. Zaripova R.M., Ilyina V.P., Naumova G.A., Selezneva V.E., Shammazova A.M. and other researchers. And also, a broad review of the literature devoted to the development of methods for calculating underground pipelines presented in the works of V.A. Chechelov, A.V. Yavarova, G.S. Kolosova, V.V. Kuroedov [1-11, 18,19].

Essentially, when calculating SSS and stability of pipelines, the influence of soil is taken into account indirectly. In this regard, this work implements an algorithm for calculating the stress-strain state of a soil environment within the diameter of a large cross-section pipe. It is assumed that the pipe is laid in a trench. Soil is considered as an elastic medium that obeys Hooke's linear law in the case of a flat deformable state. The solution is based on the variation-difference method (VDM) [12-17, 20].

Methods

We consider an orthotropic elastic medium that satisfies Hooke's linear law, which in the case of a plane deformed state is taken in the form:

$$\begin{aligned}\sigma_{rr} &= a_{11}\varepsilon_{rr} + a_{12}\varepsilon_{\varphi\varphi} \\ \sigma_{\varphi\varphi} &= a_{21}\varepsilon_{rr} + a_{22}\varepsilon_{\varphi\varphi} \\ \sigma_{r\varphi} &= 2a_{66}\varepsilon_{r\varphi}\end{aligned}\tag{1}$$

Where are r, φ, z – the axes of the cylindrical coordinate system.

The solution is based on the principle of minimum functional of total potential energy in Lagrange form:

$$\int \frac{1}{2} [a_{11}u_r^2 + a_{22}\left(\frac{u}{r} + \frac{1}{r}vu\right)^2 + 2a_{12}u_r\left(\frac{u}{r} + \frac{1}{r}vu\right) + a_{66}\left(vr - \frac{v}{r} + \frac{vu}{r}\right)^2] \tag{2}$$

The coefficients are a_{ij} ($i, j = 1, 2$), a_{66} – appropriately expressed through independent elastic physical and mechanical characteristics: E_r, E_φ, E_z – Young's modulus, $C_{r\varphi}$ – shear modulus, $\nu_{r\varphi}, \nu_{\varphi z}, \nu_{zr}$ – Poisson's ratios

$$a_{11} = \frac{E_r(1 - \nu_{\varphi z} \nu_{z\varphi})}{k};$$

$$a_{12} = \frac{E_r(\nu_{r\varphi} - \nu_{rz} \nu_{z\varphi})}{k};$$

$$a_{21} = \frac{E_\varphi(\nu_{\varphi r} - \nu_{zr})}{k};$$

$$a_{22} = \frac{E_\varphi(1 - \nu_{rz} \nu_{zr})}{k};$$

$$a_{66} = \frac{1}{C_{r\varphi}};$$

$$k = 1 - \nu_{rz} \nu_{zr} - \nu_{\varphi z} \nu_{z\varphi} - \nu_{r\varphi} \nu_{\varphi r} - \nu_{r\varphi} \nu_{\varphi z} \nu_{zr} - \nu_{rz} \nu_{z\varphi} \nu_{\varphi r}$$

In the particular case of an isotropic medium, it is enough to equate the a_{ij} ($i, j = 1, 2$), a_{66} – Young's modulus and Poisson's ratios in the expressions for and appropriately calculate the shear modulus $C_{r\varphi}$, through which the coefficient is a_{66} determined. The external load (weight of the medium, pressure along the surface) naturally enters into the expression of the functional (2).

Numerical solution method. Expression (2) regarding continuous displacement values is replaced by a finite-difference analogue. To do this, a finite-difference (f.d.) mesh is introduced (Fig. 1), and approximating f.r.s are introduced for displacements and their derivatives along coordinate directions. Ratios averaged over cells of the k.-r. mesh

$$(u_r)_{i=\varphi} = \frac{u_{k-1,l} + u_{kl} - u_{k-1,l+1} - u_{k,l+1}}{2(r_{kl} - r_{k,l+1})}$$

$$(u_\varphi)_{i=\varphi} = \frac{u_{kl} + u_{k,l+1} - u_{k-1,l} - u_{k-1,l+1}}{2(r_{kl} - r_{k-1,l+1})(\varphi_{kr} - \varphi_{k-1,l+1})} \quad (3)$$

K.-r. an analogue of the Lagrange functional can be represented in the form

$$\mathcal{O}(u_{kl}, v_{kl}) = \sum_i (W_i S_i - \mathcal{O}_i^b) \quad (4)$$

Where are u_{kl}, v_{kl} – the values of displacements in the nodes of the k.-r. grids in r, φ directions accordingly. W_i – elastic potential of the cell, i – work of external \mathcal{O}_i^b – forces applied to the cell, S_i – area of the cell. The summation in (4) is carried out over all cells of the c.r. a grid (k-numbers of radii in the direction φ, l) of nodes at k radii in the opposite direction r .

Minimum (4) is determined based on the Euler-Ostrogradsky relations.

$$\frac{\partial \mathcal{E}(u_{kl}, v_{kl})}{\partial u_{kl}} = 0$$

$$\frac{\partial \mathcal{E}(u_{kl}, v_{kl})}{\partial v_{kl}} = 0 \quad (5)$$

Where are u_{kl}, v_{kl} – the unknown displacement values?

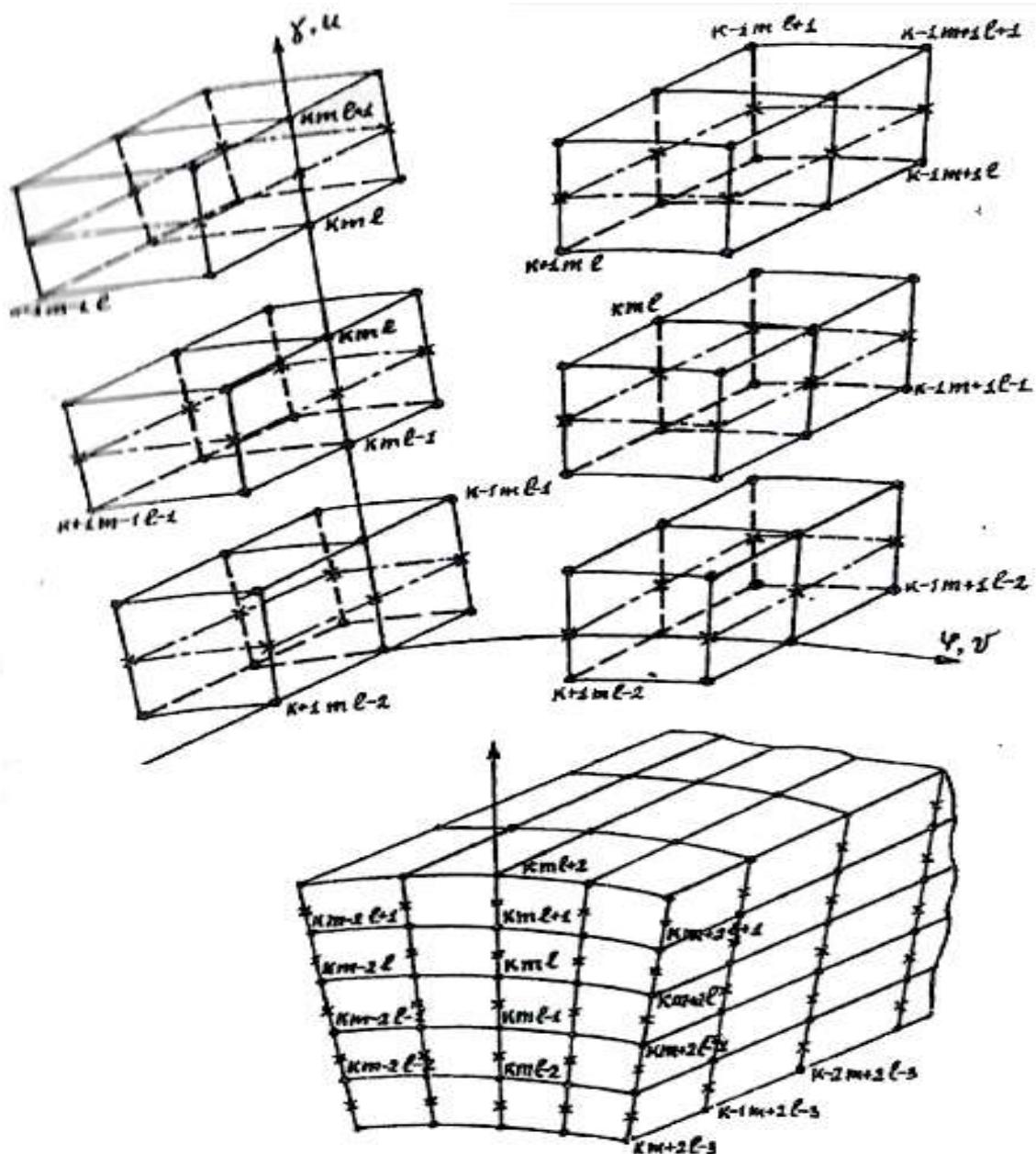


Fig.1. Spatial finite difference grid

Solution algorithm. Based on (2), assuming that the displacements and their derivatives are

$$\frac{\partial W_i}{\partial u_{kl}}, \frac{\partial W_i}{\partial v_{kl}}$$

determined by (3), expressions for $\frac{\partial W_i}{\partial u_{kl}}, \frac{\partial W_i}{\partial v_{kl}}$ can be represented as:

$$\begin{aligned} \frac{\partial W_i}{\partial u_{kl}} = & \left(a_{11} \frac{\partial u_r}{\partial u_i} + a_{12} \frac{1}{r} \frac{\partial u}{\partial u_i} \right) u_r + \left(a_{22} \frac{1}{r^2} \frac{\partial u}{\partial u_i} + a_{12} \frac{1}{r} \frac{\partial u_r}{\partial u_i} \right) u + \\ & + a_{66} \frac{1}{r^2} \frac{\partial u_\varphi}{\partial u_i} u_\varphi + a_{66} \frac{1}{r} \frac{\partial u_\varphi}{\partial u_i} v_r + \left(a_{22} \frac{1}{r^2} \frac{\partial u}{\partial u_i} + a_{12} \frac{1}{r} \frac{\partial u_r}{\partial u_i} \right) v_\varphi - \\ & - a_{66} \frac{1}{r^2} \frac{\partial u_\varphi}{\partial u_i} v; \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial W_i}{\partial v_{kl}} = & a_{12} \frac{1}{r} \frac{\partial v}{\partial u_i} u_r + a_{66} \left(\frac{1}{r} \frac{\partial v_r}{\partial v_i} - \frac{1}{r^2} \frac{\partial v}{\partial v_i} \right) u_\varphi + a_{22} \frac{1}{r^2} \frac{\partial v_\varphi}{\partial v_i} u + \\ & + \left(a_{66} \frac{\partial v_r}{\partial v_i} - a_{66} \frac{1}{r} \frac{\partial v}{\partial v_i} \right) v_r + a_{22} \frac{1}{r^2} \frac{\partial v_\varphi}{\partial v_i} v_\varphi + \left(a_{66} \frac{1}{r^2} \frac{\partial v}{\partial v_i} - a_{66} \frac{1}{r} \frac{\partial v_r}{\partial v_i} \right) v. \end{aligned}$$

Here $(u)_i, (r)_i, (u_r)_i, \dots$ – the averaged values of the functions and derivatives of the cells of the k.-r. grids. The index is i omitted throughout for brevity, and the double index is kl replaced by the index i .

The coefficients in (6), with averaged displacements and their derivatives within i – each cell, are determined by the physical and mechanical characteristics of the cell material and its geometry. To calculate these coefficients, $\mathcal{A}(N, M, ij)$, where N, M – the number of the node located in relation to the cell as shown in Fig. 1. The same figure defines the numbering of nodes inside the cell. Depending on which number inside the cell corresponds to the node in which the movements vary, the type of cell is u_{kl}, v_{kl} determined. Suppose the displacements corresponding to the first node inside the cell vary, then values are N, u, M assigned accordingly $k, l - 1$. (see Fig. 2).

In general, a node kl can surround up to four cells. (Fig. 3). Depending on the cell type, a value is assigned to the parameter j in $\mathcal{A}(N, M, ij)$.

In the case when the node belongs to a triangular cell (boundary cells in Fig. 1). For the purpose of a unified record of operators, an additional node was introduced, marked.

This made it possible to solve the problem by using an algorithm implemented earlier in works in which the obtained mesh q.r. the equations were solved using the algorithm [7]. As in [7], the solution to the problem of determining the displacements of an elastic medium is carried out using the direct Gaussian elimination method when solving the system of corresponding linear equations (5). The stresses were calculated using formulas (1), in which the plane strain components are:

$$\begin{aligned}\varepsilon_r &= \frac{\partial u}{\partial r} = u_r; \quad \varepsilon_{\varphi\varphi} = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \varphi} = \frac{u}{r} + \frac{1}{r} v_\varphi; \\ \varepsilon_{r\varphi} &= \frac{1}{2} \left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \frac{\partial u}{\partial \varphi} \right) = \frac{1}{2} \left(v_r - \frac{v}{r} + \frac{1}{r} v_\varphi \right) \\ \varepsilon_{zz} &= \varepsilon_{rz} = \varepsilon_{\varphi z} = 0.\end{aligned}\tag{7}$$

In this case, the displacements and their derivatives and the corresponding functions were replaced by approximating relations [13].

Results and Discussion

The results obtained are presented in comparison with experimental data regarding the distribution of soil pressure along the perimeter of the pipe presented in [13, 16, 20]. The qualitative picture of the obtained stress-strain state distribution of an elastic medium is consistent with that presented in [7]. In the case of a flexible pipe, the pressure along the perimeter of the flexible pipe is equalized [7, 9, 10], which was also confirmed by experiments. The presented results relate to the calculation of pipelines and relate mainly to checking the reliability of the results obtained based on the implemented algorithm in comparison with the available experimental data on the distribution of soil pressure along the perimeter of the pipe [7].

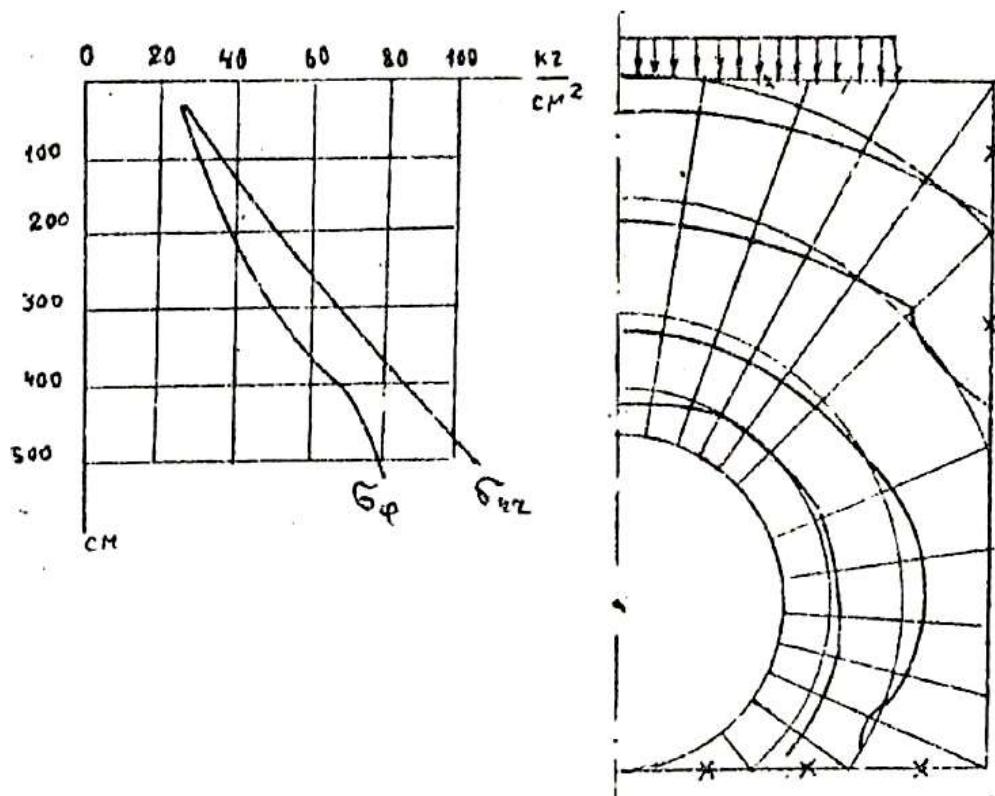


Fig.2. a) k.-r. mesh of flat section of soil and circular shell; b) distribution of maximum radial and circumferential soil stresses along the perimeter of the cylindrical shell

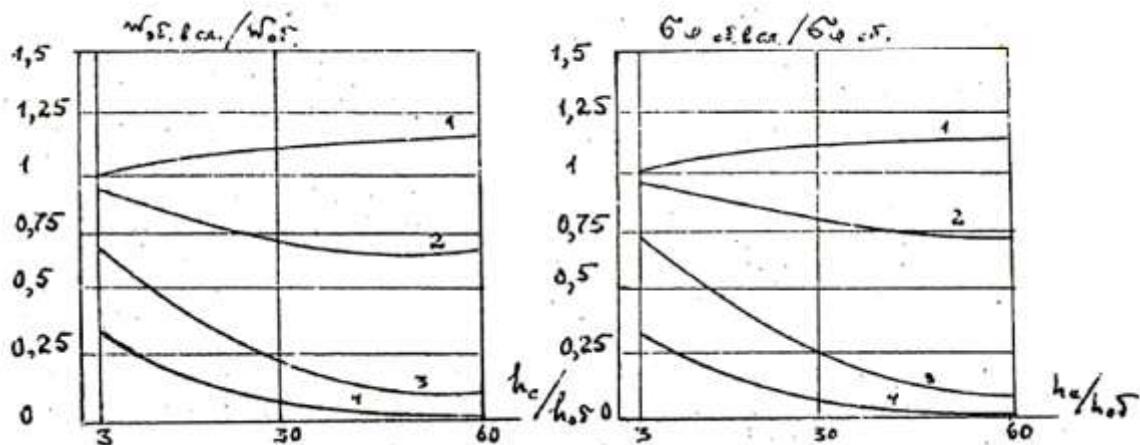


Fig.3. a) relative deflection of the shell interacting with the external elastic layer depending on the rigidity and thickness of the soil; b) relative hoop stress in the shell depending on the thickness of the soil.

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